Poynting Flux out of Rotating Black Hole and Accretion Flow through Force-Free Magnetosphere*

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The basic features of the Poynting flux from the horizon and the ergosphere of a black hole and from the accreting flow onto a black hole are discussed for the force-free magnetosphere. The accretion flow dominated by the Poynting flux is discussed and the possible Poynting flux from the equatorial plane inside the ergosphere is discussed.

PACS numbers: 97.10.Gz, 97.60.Lf
Keywords: Poynting flux, Black hole, Accretion flow, Force-free magnetosphere

I. INTRODUCTION

The studies of the explosive processes like SNe and GRBs have been revealing the evidences of compact objects such as neutron star and/or black hole at the center of explosive events. And it is likely that massive black holes are responsible for the activities of AGN. The existence of a black hole leads to the idea that the energy sources for powering violent astrophysical phenomena like AGN and GRB are likely to be the gravitational binding energy of the accreting material onto a black hole and the rotational energy of a black hole itself[1, 2, 3, 4]. It has been also expected that the strong and large scale magnetic field configuration can be formed around the compact objects at the centers, particularly for the late stage of the core-collapses of rapidly rotating and strongly magnetized stars. It is then natural to suppose a simple physical situation in which the energy of the accreting material and the rotational energy of a black hole can be extracted out through the magnetic field lines in the form of the Poynting flux.

The Poynting flux has been considered to be a viable route to transport the energy and angular momentum along the magnetic field lines and one of the efficient ways to tap the black hole's rotational energy using magnetic field lines anchored on the horizon[1] supported by the external current. The relevance of using the Poynting flux in describing the powerful and highly collimated astrophysical jets observed in AGN and quasars has been suggested long time ago [3, 4] and many interesting works have been developed. One of the characteristics of the Poynting flux is that it carries very little baryonic component compared to the hydrodynamic flow. This property of Poynting flux is found to be consistent with the required property for powering GRB[5]. Recently numerical simulations also have begun to demonstrate the electromagnetic energy extraction [6, 7, 8] from a black hole surrounded by the force-free magnetosphere, which can be established when the electromagnetic field is strong enough with sufficient charged particles for space-charges and currents and the inertia of the plasma can be ignored. The Poynting flux in a system of black hole-accretion disk has also been studied in connection with GRBs[9, 10, 11] and it is found that the evolution of the system largely depends on the Poynting outflow from the disk[12, 13, 14, 15].

In this work, the nature of the Poynting flux from the horizon and the ergosphere of a black hole and from the accreting flow onto a black hole will be discussed. To make the discussion simpler the environment through which the Poynting flux carries energy out is assumed to be force-free with steady and axisymmetric configuration and the accreting flow is assumed to be confined on the equatorial plane. In section II, the force-free magnetosphere is introduced together with the consistent constraint and the basic features of the Poynting flux is discussed in section III. In section IV, the accretion flow dominated by the Poynting flux is discussed briefly and the possible Poynting flux from the equatorial plane inside the ergosphere is discussed in section V.

II. FORCE-FREE MAGNETOSPHERE IN KERR BACKGROUND

The force-free magnetosphere can be established when the electromagnetic field is strong enough with sufficient charged particles for space-charges and currents and the inertia of the plasma can be ignored. The role of the plasma in the force-free magnetosphere is to provide the charge and current sources for the field. The mathematical expression for the force-free limit is given by

$$F_{\mu\nu} J^\nu = 0, \tag{1}$$

where J^ν is the electromagnetic current density. In terms of the physical quantity defined by FIDO, it can be written as

$$\rho \vec{E} + \vec{J} \times \vec{B} = 0, \tag{2}$$

from which one can also observe that

$$\vec{E} \cdot \vec{B} = 0, \tag{3}$$

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*This paper was presented at APCTP Winter School on Black Hole Astrophysics 2006.
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which implies that the electric potential is conserved along the magnetic field lines. It should be noted that the force-free condition is not identical to the ideal MHD condition,

$$F_{\mu\nu}u^{\nu} = 0, \tag{4}$$

or equivalently

$$\vec{E} + \vec{v} \times \vec{B} = 0, \tag{5}$$

where u^{ν} is the four velocity of the plasma.

The force-free requirement should be consistent with the condition that there is no local frame in which a pure electric field can be seen[16, 17]. In other words, the invariant

$$B^2 - E^2 = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} \geq 0, \tag{6}$$

has to be always checked. On the other hand, when one can find a region where Eq.(6) can not be consistent it is very likely that it is the signature of a break-down of force-free condition. Hence it can be a very useful method to find out the non-force-free region (most likely a part of the region) even without knowing the detailed knowledge of the plasma around.

In the steady and axisymmetric case around the Kerr black hole, we get from the the force-free condition, Eq.(1),

$$\vec{E} = -\frac{\bar{\omega}}{\alpha}(\Omega_F + \beta)\ e^{\hat{\phi}} \times \vec{B}^p. \tag{7}$$

Ω_F, which is defined by $dA_0 = -\Omega_F dA_\phi$, is the angular velocity of the magnetic surface that rotates rigidly in an axisymetric and stationary state [18]. Throughout this work, the background metric is assumed to be the Kerr metric[19]. Using the Boyer-Lindquist coordinates[20] in the natural unit $G = c = 1$, the non vanishing components are given by

$$\begin{aligned} g_{00} = -(\alpha^2 - \varpi^2\beta^2), \quad g_{0\phi} = g_{\phi 0} = \varpi^2\beta, \\ g_{rr} = \frac{\rho^2}{\Delta}, \quad g_{\theta\theta} = \rho^2, \quad g_{\phi\phi} = \varpi^2, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \alpha &= \frac{\rho\sqrt{\Delta}}{\Sigma}, \quad \beta = -\frac{2aMr}{\Sigma^2}, \quad \tilde{\omega} = \frac{\Sigma}{\rho}\sin\theta, \\ \Delta &= r^2 + a^2 - 2Mr, \quad \rho^2 = r^2 + a^2\cos^2\theta, \\ \Sigma^2 &= (r^2 + a^2)^2 - a^2\Delta\sin^2\theta. \end{aligned} \tag{9}$$

Using Eq.(7), the invariant $B^2 - E^2$ can be written as

$$B^2 - E^2 = -f(\Omega_F, r, \theta)(B^p)^2 + (B^{\hat{\phi}})^2, \tag{10}$$

where B^p is the poloidal component of the magnetic field and

$$\begin{aligned} f(\Omega_F, r, \theta) &\equiv \Omega_F^2 g_{\phi\phi} + 2\Omega_F g_{0\phi} + g_{00} \\ &= -\left[\frac{\alpha^2 - \varpi^2(\Omega_F + \beta)^2}{\alpha^2}\right]. \end{aligned} \tag{11}$$

As a simple example, consider a region very near to the horizon, $\alpha \to 0$. Then Eq.(10) can be written as

$$B^2 - E^2 = -\left[\frac{\varpi^2(\Omega_F - \Omega_H)^2}{\alpha^2}\right](B^p)^2 + (B^{\hat{\phi}})^2. \tag{12}$$

One can see that without the toroidal component of the magnetic field, $B^{\hat{\phi}}$, the above equation becomes negative and the force-free condition breaks down.

The force-free condition is assumed for the plasma with a strong magnetic field, which is sufficiently tenuous that the plasma exerts no force on the magnetic field. If the whole region is force-free, there is no place to extract energy for the Poynting flux. We suppose a Poynting flux as a viable route to transport the energy and angular momentum along the magnetic field lines. Then there should be a region where matters exerts forces on the magnetic field such that they convey the energy and angular momentum along the field lines in the form of Poynting flux. For the Poynting flux out of accretion disk, it is naturally expected that the force-free condition holds only for the magnetosphere not on the disk.

III. BASIC FEATURES OF POYNTING FLUX

To discuss the basic feature of the Poynting flux, suppose an artificial surface at large radius which encloses the black hole and accretion disk for example. The electromagnetic field near the surface is described by the the Maxwell equation,

$$F^{\mu\nu}_{;\nu} = 4\pi J^{\mu}. \tag{13}$$

The conservation of bulk current density should be modified since the bulk current density has a discontinuity on this arbitrary surface[21]. To ensure the conservation of the current with this boundary, the total conserved current density $\mathcal{J}^{\mu}$ should be the sum of the bulk current density and the surface current density j^{μ}:

$$\mathcal{J}^{\mu} = J^{\mu} + j^{\mu}, \tag{14}$$

where the effective surface current density are determined using the current conservation law

$$\mathcal{J}^{\mu}{}_{;\mu} = 0. \tag{15}$$

As an example, let us consider a spherical surface for an axisymmetric and steady case. One can have

$$\sigma = \frac{1}{4\pi}E^r, \quad j^{\theta} = -\frac{1}{4\pi}B^{\phi}, \quad j^{\phi} = \frac{1}{4\pi}B^{\theta}. \tag{16}$$

If the radius of the arbitrary surface is located at far enough distance from the central object, then we can use a flat space-time geometry. Then the energy flux of electromagnetic field is given by

$$\mathcal{E}^r = \frac{1}{4\pi}E^{\theta}B^{\phi}, \tag{17}$$

which is the radial component of the Poynting flux. On the arbitrary surface using Eq.(16), the Poynting flux can be written as

$$\mathcal{E}^r = -j^\theta E^\theta. \tag{18}$$

One can note that for a non-vanishing Poynting flux the force-free condition cannot be maintained on the boundary, $\vec{j} \cdot \vec{E} \neq 0$. It corresponds to the 'effective battery if it is positive (or 'ohmic dissipator' if negative). Similarly the z-component of the angular momentum flux can be written by

$$-\varpi B^\phi B^r = -\frac{1}{4\pi}\varpi j^\theta B^r, \tag{19}$$

where Eq.(16) is used and ϖ is a cylindrical radius. It corresponds to the effective 'torque' in z-direction exerted by the surface current although it is force-free in the original setting. This observation implies that the non-force-free nature of the system sitting at the center inside the surface can be manifested on the arbitrary boundary surface outside.

Now put the arbitrary surface near to the gravitating object with the smallest radius but big enough to enclose the energy source while keeping outside to be force-free:

$$F_{\mu\nu} J^\nu = 0. \tag{20}$$

We need a fully relativistic treatment. For Kerr geometry, the energy flux[22] is given by

$$\mathcal{E}^{\hat{r}} = \frac{\alpha}{4\pi} E^{\hat{\theta}} B^{\hat{\phi}} + \frac{\beta\varpi}{4\pi} B^{\hat{\phi}} B^r. \tag{21}$$

Using Eq.(7), for a rigidly rotating magnetic field line with angular velocity Ω_F in a force-free environment up to the boundary surface, Eq.(21) can be written by

$$\mathcal{E}^{\hat{r}} \; = \; \frac{\alpha}{4\pi} \frac{\Omega_F}{(\Omega_F + \beta)} E^{\hat{\theta}} B^{\hat{\phi}}, \tag{22}$$

which shows a typical form of Poynting flux.

Using the effective currents defined above we can obtain

$$\mathcal{E}^{\hat{r}} = \alpha j^\theta E^{\hat{\theta}} + \beta\varpi j^{\hat{\theta}} B^{\hat{r}}. \tag{23}$$

We can note that the non-force-free nature of the source inside for the Poynting flux outside is represented by non-vanishing $j^{\hat{\theta}}$ which 'flows' across the magnetic field lines on the boundary surface. Basically if the whole region is force-free then there is no way to get a Poynting flux: no energy source. Hence the magnetic field lines should be anchored onto a non-force-free region from which the Poynting flux can carry out the energy. However the detailed nature of non-force-free region can only be understood in the frame work of the relativistic magnetohydrodynamics.

IV. POYNTING FLUX DOMINATED ACCRETION FLOW

We consider a system of a rotating black hole and a two-dimensional accretion disk, which is assumed to be on the equatorial plane in the background metric of a Kerr black hole and the accretion is supposed to be driven by the Poynting flux[23]

The stress-energy tensor is decomposed into two parts, the matter part, $T^{\mu\nu}_D$, and the electromagnetic part, $T^{\mu\nu}_{EM}$:

$$T^{\mu\nu} = T^{\mu\nu}_D + T^{\mu\nu}_{EM}. \tag{24}$$

In general the stress-energy tensor for the accretion disk is determined by mass density, internal energy, pressure, viscosity, radiative transfer and etc. [24]. Since we are interested in the accretion dominated by the Poynting flux, the two-dimensional accretion disk is assumed to be non-viscous, cool, and non-radiative such that the matter part is given by

$$T^{\mu\nu}_D = \rho_m u^\mu u^\nu, \tag{25}$$

where ρ_m is the rest-mass density and u^μ is the four velocity of the accreting matter. It is also assumed that there is a negligible mass flow in the direction perpendicular to the disk: $u^\theta = 0$. We can obtain the energy flux and the angular momentum flux given by

$$\mathcal{E}^\mu_D = -\rho_m u_0 u^\mu, \quad \mathcal{L}^\mu_D = \rho_m u_\phi u^\mu. \tag{26}$$

For an idealized thin disk on the two-dimensional plane we take

$$\rho_m = \frac{\sigma_m}{\rho}\delta(\theta - \pi/2), \tag{27}$$

where σ_m is the surface rest-mass density. The rate of the rest-mass flow crossing the circle of radius r defines the mass accretion rate $\dot{M}_+$ by

$$\dot{M}_+ = -2\pi\sigma_m \rho u^r. \tag{28}$$

It is identical to the one derived in [24] and [25], in which the vertical structures of the disk are integrated. The stationary accretion flow implies that $\dot{M}_+$ is r-independent. Using Eq.(26) the radial flow of the energy and angular momentum of the matter at r can be given by $u_0 \dot{M}_+$ and $-u_\phi \dot{M}_+$ respectively.

The horizon of a black hole is a mathematically well defined surface, on which the appropriate surface current density has been studied in depth[18]. On the other hand, the surface of the physical accretion disk is not expected to have any sharp boundary and various shapes have been suggested depending on the properties of the accretion flow. In this work, however, we assume a simplified accretion disk, which is vertically squeezed down to the equatorial plane, a two-dimensional accretion disk. Then

we can associate the surface current density in Eq.(14) in a simple way,

$$j^{\mu} = \frac{1}{4\pi}(F_{+}^{\theta\mu} - F_{-}^{\theta\mu})\delta(\theta - \pi/2), \tag{29}$$

which are nothing but Gauss' law and Ampere's law as given by

$$\sigma_e = -\frac{1}{2\pi}E_{+}^{\hat{\theta}}, \quad K^{\hat{r}} = -\frac{1}{2\pi}B_{+}^{\hat{\phi}}, \quad K^{\hat{\phi}} = \frac{1}{2\pi}B_{+}^{\hat{r}}, \tag{30}$$

where σ_e and K^i are surface charge and surface current density(spatial) respectively and the reflection symmetry with respect to the equatorial plane, $F_{+}^{\theta\mu} = -F_{-}^{\theta\mu}$, is used. The surface current densities are responsible for the Poynting flux from the disk. Hence one can consider the singular equatorial plane with non-vanishing surface currents manifests the non force-free nature of the accretion flow driven by Poynting flux.

The energy and the angular momentum of the disk can be carried out by Poynting flux along the magnetic field lines which are anchored on the disk. In our simplified model, it is the main driving force for the accretion flow.

Using the Killing vector in t-direction, $\xi^{\mu} = (1, 0, 0, 0)$, we can define the energy flux $\mathcal{E}^{\mu}$ from the stress-energy tensor of the electromagnetic field, $T_{EM}^{\mu\nu}$,

$$\mathcal{E}_{EM}^{\mu} = -T_{EM}^{\mu\nu}\xi_{\nu}, \tag{31}$$

where

$$T_{EM}^{\mu\nu} = \frac{1}{4\pi}(F^{\mu}_{\ \rho}F^{\nu\rho} - \frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}). \tag{32}$$

For the steady and axisymmetric case in this work, $E^{\phi} = 0$, and we get the total energy flux of electromagnetic field in θ-direction, $\mathcal{E}_{D}^{\theta}$, given by

$$\mathcal{E}_{D}^{\theta} \ = \ \frac{1}{\rho}[\alpha K^{\hat{r}}E_{+}^{\hat{r}} - \beta\varpi K^{\hat{r}}B_{+}^{\hat{\theta}}]. \tag{33}$$

The first term in Eq.(33) can be considered to be an ohmic interaction, $\vec{E}\cdot\vec{K}$. For the current in the direction of the electric field tangential to the disk there might be an energy dissipation into the disk surface. This is what one can expect on the black hole horizon[18]. However for the current in the opposite direction this term corresponds to the electro-motive force and it is the case for the accretion disk on the equatorial plane discussed by Blandford and Znajek[1, 4]as well as in this work. The second term can be interpreted as a magnetic braking power on the rotating body with an angular velocity $\omega = -\beta$.

Using the Killing vector in ϕ-direction for the axial symmetric case, $\eta^{\mu} = (0, 0, 0, 1)$, we can get also the total flux of the angular momentum in θ-direction, $\mathcal{L}_{D}^{\theta}$,

$$\mathcal{L}_{D}^{\theta} = -\frac{\varpi}{2\pi\rho}B_{+}^{\hat{\theta}}B_{+}^{\hat{\phi}} = \frac{1}{\rho}\varpi K^{\hat{r}}B_{+}^{\hat{\theta}}, \tag{34}$$

which is nothing but a magnetic torque exerted on the surface current density K^{r}.

Now the Poynting power measured at infinity is given by

$$P_D \ = \ -\int drr(-\alpha E_{+}^{\hat{r}}B_{+}^{\hat{\phi}} + \beta\varpi B_{+}^{\hat{\phi}}B_{+}^{\hat{\theta}}), \tag{35}$$

and the rate of angular momentum transfer, P_D^L, out of the accretion flow is given by

$$P_D^L \ = \ \int drr\varpi B_{+}^{\hat{\phi}}B_{+}^{\hat{\theta}}. \tag{36}$$

A. Accretion Equations and Stream Equation

The dynamics of an accretion flow is determined by the conservation equation of the stress-energy tensor:

$$T^{\mu\nu}_{\ \ ;\mu} = 0. \tag{37}$$

From the conservation of stress-energy tensor, one can obtain the accretion equations [26] driven by the Poynting flux in a force-free magnetoshpere given by

$$(\partial_r u_0)\dot{M}_{+} + r\varpi\Omega_F B_{+}^{\hat{\phi}}B_{+}^{\hat{\theta}} = 0, \tag{38}$$

$$(\partial_r u_{\phi})\dot{M}_{+} - r\varpi B_{+}^{\hat{\phi}}B_{+}^{\hat{\theta}} = 0, \tag{39}$$

$$\frac{\dot{M}_{+}}{2\pi r^2}g^{rr}\frac{u^0}{u^r}(\partial_r u_{\phi})\left[\Omega_D + \frac{\partial_r u_0}{\partial_r u_{\phi}}\right] - \frac{\sqrt{\Delta}}{2\pi r^2}B_{+}^{\hat{r}}B_{+}^{\hat{\theta}}\left(\frac{\varpi^2}{\alpha^2}\Omega_F^2 - 1\right) = 0, \tag{40}$$

where the angular velocity of the disk is given by $\Omega_D = u^{\phi}/u^0$. Eqs.(38) and (39) correspond to the energy and angular momentum conservation respectively in the stationary and axisymmetric setting in this work. In Eq.(38) the radial change of the energy flux of accreting matter, $-u_0\dot{M}_{+}$, is balanced by the Poynting flux in theta direction[22], $\mathcal{E}_{D}^{\theta}$. Similarly the radial change of the angular momentum flux, $u_{\phi}\dot{M}_{+}$ is balanced by $\mathcal{L}_{D}^{\theta}$. The radial equation Eq.(40) essentially determines the orbital motion of the disk. In the absence of external fields in Eq.(40), the angular velocity is determined as

$$\Omega_D = -\frac{\partial_r u_0}{\partial_r u_{\phi}}, \tag{41}$$

which is one of the characteristics of the Keplerian orbit. Hence deviations of Ω_D from the Keplerian one are naturally expected in the Poynting flux dominated accretion disk.

It should be noted the force-free condition holds only for the magnetosphere not on the disk. In fact the driving force for an accretion flow on the disk is the magnetic braking in Eqs.(38) and(39). Then Eq.(35) becomes

$$P_D \ = \ \int drr(\varpi\Omega_F B_{+}^{\hat{\theta}}B_{+}^{\hat{\phi}}). \tag{42}$$

From Eqs.(38) and (39), one obtains an interesting relation

$$\frac{\partial_r u_0}{\partial_r u_\phi} = -\Omega_F. \tag{43}$$

which has no explicit dependence on the field configuration. It seems to imply that Ω_F is determined essentially by the dynamics of the disk. However one should note that the dynamics itself is governed not only by the gravity but also by the electromagnetic field as well.

The configuration of the ordered magnetic field around the accretion disk and the Poynting outflow from the disk has been discussed both in analytical and numerical studies[27, 28, 29, 30]. For the strong enough electromagnetic field around the compact object, the force-free magnetosphere can be established. In the non-relativistic formulation, Blandford[4] suggested an axisymmetric and stationary electromagnetic field configuration around an accretion disk on the equatorial plane. The poloidal field configuration for a black hole is known to satisfy a second order elliptic differential equation called Grad-Shafranov equation[31] or a stream equation[32] for the stream function Ψ and current I. The poloidal and toroidal components of the magnetic field can be written in terms of Ψ and I respectively:

$$\vec{B}^P = \frac{1}{2\pi\varpi}\nabla\Psi \times e^{\hat{\phi}}, \quad B^{\hat{\phi}} = -\frac{2I}{\varpi\alpha}. \tag{44}$$

Possible types of solutions in a force-free magnetosphere has been discussed recently in the relativistic formulation[33, 34]. To calculate the Poynting flux we need to know the configuration of the magnetic field which is consistent with the accretion equations. And the problem is reduced to solve the four coupled equations, three accretion equations and a Grad-Shafranov equation(stream equation).

B. Example: Numerical Solutions in Schwarzschild Background

The coupled differential equations in a Kerr geometry in the previous section is not easy to solve. To show some examples of the solutions we try a numerical solution[35] in a simpler background, Schwarzschild background. In the Schwarzschild background, the stream equation is given by

$$\begin{aligned} &\partial_r\left\{\left(1-\frac{2M}{r}\right)\partial_r\Psi\right\} + \frac{\sin\theta}{r^2}\partial_\theta\left(\frac{1}{\sin\theta}\partial_\theta\Psi\right) \\ &- \Omega_F\sin^2\theta\partial_r\left(r^2\Omega_F\partial_r\Psi\right) - \frac{\Omega_F}{1-\frac{2M}{r}}\sin\theta\partial_\theta\left(\sin\theta\Omega_F\partial_\theta\Psi\right) \\ &= -\frac{16\pi^2 I\frac{dI}{d\Psi}}{\left(1-\frac{2M}{r}\right)}. \end{aligned} \tag{45}$$

In general there is no known analytic forms for the solutions. In the limiting case when Ω_F and I vanishes, one of the solutions, Ψ_0, suggested by Blandford and Znajek [1] is given by

$$\Psi_0 = \pi C X, \tag{46}$$

where

$$X \equiv r(1\mp\cos\theta) + 2M(1\pm\cos\theta)\{1-\log(1\pm\cos\theta)\}. \tag{47}$$

For non-vanishing Ω_F and I, we consider a solution for which the shape of the magnetic surface are the same as the magnetic surface defined by Ψ_0[36]. Thus Ψ, I and Ω_F are assumed to depend on X. We suppose that the derivative of Ψ has the same form as in the flat background, Eq.(46). That is, we take an Ansatz such that

$$\frac{d\Psi}{dX} = \frac{\pi C}{(1+\Omega_F^2 X^2)^{1/2}}. \tag{48}$$

Four basic equations governing the accretion flow under the influence of paraboloidal-type configuration are solved numerically[35]. The radial variations of u_0, u_ϕ, I and Ω_F on the disk are basically functions of the accretion rate and the strength of the magnetic field. Numerical calculations show that the angular velocities of the magnetic field lines(Ω_F) are different from either the Keplerian angular velocity(Ω_K) or the disk angular velocity(Ω_D). It implies that the two-dimensional approximation of the perfect conducting disk, for which $\Omega_F = \Omega_D$, may not be implemented particularly with the paraboloidal type configuration. As expected in the previous section for the velocity of the magnetic field line $v_F(= r\Omega_F)$ less than the speed of light, Ω_F is found to be larger than Ω_D.

It is also found that the strength of the magnetic field is increasing as r goes near the inner edge. Since the magnetic field as well as the surface current are increasing as r gets smaller, it is naturally expected that the Poynting flux increases substantially as r approaches to the center . Although the numerical calculation does not go beyond $r < 6M$, it may indicate a possible electromagnetic jet structure near the inner edge of the disk.

V. POYNTING FLUX FROM ERGOSPHERE

On the horizon inside the ergosphere, we can make use of the Znajek's boundary condition[16],

$$B^{\hat{\phi}} = \frac{\varpi(\Omega_F - \Omega_H)}{\alpha}B^{\hat{r}}, \tag{49}$$

and we get

$$B^2 - E^2 = -\left[\frac{\varpi^2(\Omega_F-\Omega_H)^2}{\alpha^2}\right](B^{\hat{\theta}})^2. \tag{50}$$

It implies that for the magnetosphere with $B^\theta = 0$ on the horizon the force-free nature can be justified and the region where the electromagnetic field lines can exert nonzero force should be inside horizon. As discussed in section III, we can define a non-zero surface current on the

horizon boundary such that the rotation of black hole can be slowed down or speeded up and we get the Poynting flux[1]. Using Eq.(22), the rate of energy extraction at infinity can be evaluated using the electromagnetic field on the horizon,

$$P_{hole} = -\int_{r=r_H} \frac{1}{4\pi}\Omega_F(\Omega_F - \Omega_H)\varpi^2(B^{\hat{r}})^2 d\theta d\phi. \quad (51)$$

It is to be noted that the force-free condition might not be maintained on the horizon for $B^{\hat{\theta}} \neq 0$. It may corresponds to the case when the force-free condition is invalid in the vicinity of horizon[39], which, however, depends on the details of the physical properties of the plasma.

We will explore whether similar analysis can be applied to the magnetic field lines threading on the equatorial plane inside the ergosphere[38]. The overall environment and physical processes are assumed to be axisymmetric and steady in the background of Kerr geometry. The effects on the geometry due to the energy density of the electromagnetic field and the accreting material are assumed to be negligible. In this work we assume a very thin accretion flow on the equatorial plane. Let us suppose the force-free relation, $F^{\mu\nu}J_\nu = 0$, holds up to the equatorial plane to see under what condition $B^2 - E^2 > 0$ is violated. Because of the reflection symmetry in the equatorial plane, there is no current crossing the equatorial plane and $B^\phi = 0$ on the equatorial plane. Then we get

$$B^2 - E^2 \ = \ -f(\Omega_F, r, \theta = \pi/2)\frac{(B^p)^2}{\alpha^2}, \quad (52)$$

where $f = 0$ determines the light surfaces[40]. For a given r, the sign of $B^2 - E^2$ is then determined by f. Hence the condition for force-free requirement , $f < 0$, determines the acceptable range of the angular velocity, $\Omega_F(r)$, of the field lines which cross the equatorial plane at r. One obtains

$$\Omega_-(r) \leq \Omega_F(r) \leq \Omega_+(r), \quad (53)$$

where

$$\Omega\pm = \Omega_{ZAMO} \pm \frac{\alpha}{\varpi}, \ \ \Omega_{ZAMO} = -\beta. \quad (54)$$

For such magnetic field lines which rotates with $\Omega_-(r) \leq \Omega(r) \leq \Omega_+(r)$, we cannot expect any Poynting flux since it is simply force free. In other words, we cannot get non-vanishing $B^{\hat{\phi}}$ for the Poynting flux. It is because for the force-free magnetosphere, $B_T = \varpi\alpha B^{\hat{\phi}}$ is constant on the magnetic surface and therefore $B^{\hat{\phi}}$ should remain to be zero not only on the equatorial plane but also off equatorial plane along the field line.

However for the field line with the angular velocity smaller than Ω_-, the region on the equatorial plane cannot be consistent with the force-free requirement. Hence the magnetic field line with $\Omega_F < \Omega_-$ can exert a force on the non-force-free region near the plane. And we can get a Poynting flux along such field lines, which pass through the non-force-free region. Consider a non force-free region with finite width around equatorial plane. Since it is not a force-free region, the toroidal component can be developed along the field line off the plane, although on the equator it should be zero. It reaches the boundary of force-free region with $B^\phi = B^\phi_+$, which is determined by

$$-f(\Omega, r, \theta)\frac{(B^p_+)^2}{\alpha^2} + (B^{\hat{\phi}}_+)^2 = 0. \quad (55)$$

We can guess Ω_F is not much different form Ω_-:

$$\Omega_F = \Omega_- + \delta, \quad (56)$$

where δ is determined by B^ϕ_+. It also implies that the Poynting flux out of equator is possible only inside the ergosphere since $\Omega_- = 0$ on static limit and outside the ergosphere there is no Poynting flux. It is consistent with the recent numerical simulations by Komissarov[17].

Let us consider a simple example where the non force-free region can be approximated as a thin disk, two dimensional disk on the equatorial plane, dominated by the inertia. Then we can suppose no-vanishing $B^{\hat{\phi}}$ up to the equatorial plane with discontinuity at the plane. In this boundary value problem, the surface charge, σ_e, and the current density, K^i, can be defined on the equatorial plane as in the previous section. The energy flux in θ-direction is given by Eq.(33). Inside the disk, it is not force-free and there is no reason that $B_T = \varpi\alpha B^\phi$ should be conserved along the field lines and B_T or equivalently B^ϕ develops from zero to finite value up to the disk surface. Only beyond the surface where force-free condition can be realized, B_T is conserved along the magnetic surface. The Poynting power measured at infinity is given by

$$P_{ergo} \ = \ \int_{r_H}^{r_o} dr r\varpi\Omega_F B^{\hat{\phi}}_+ B^{\hat{\theta}}_+. \quad (57)$$

It is interesting to compare the power from the disk inside the ergosphere, P_{ergo}, to the Poynting power from a black hole P_{hole}, Eq.(51). As an example we take a magnetosphere similar to that suggested by Blandford[4]. Using a set of approximations for the magnetic field and Ω_F for a numerical estimation[38], it is found that the ratio of the Poynting power from the ergosphere to that from the black hole, $P_{ergo}(\tilde{a})/P_{hole}(\tilde{a})$, is increasing as the angular momentum parameter $\tilde{a}$ increases. The ratio of the angular momentum flux, $P^{L_\phi}_{ergo}(\tilde{a})/P^{L_\phi}_{hole}(\tilde{a})$, is also increasing with $\tilde{a}$. In this simple analysis it is observed that for a maximally rotating black hole the power from the disk inside the ergosphere can be as much as 30% of the power from the black hole.

Physically P_{ergo} is a part of the gravitational energy of the particles of the negative energy orbit in a disk on the equatorial plane tapped by the magnetic field. For example, if the energy momentum tensor of the two dimensional disk is dominated by the inertia, mass density

ρ_m, we can make use of Eqs.(38) and (39). We can see that the second terms in these equations correspond to P_{ergo} and $P_{ergo}^{L_\phi}$ which depend on the energy(negative) of the orbit and the mass accretion rate. The estimation of the first terms in Eqs.(38) and (39) inside the ergosphere is an interesting subject to be studied in the future to get a more realistic estimation of P_{ergo}.

Acknowledgment

The author would like to thank Roger Blandford for discussions and the kind hospitality during his stay at KIPAC. He also thanks Dmitri Uzdensky for valuable comments. This was supported by grant No. (R01-2006-000-10651-0) from the Basic Research Program of the Korea Science & Engineering Foundation.

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[1] R. D. Blandford and R. L. Znajek,Mon. Not. Royal Astron. Soc. , **179**, 433 (1977)
[2] H. K. Lee, R. A. M. Wijers and G. E. Brown, Phys. Rep., **325**, 83 (1999)
[3] R. V. E. Lovelace, Nature, **262**, 649 (1976)
[4] R.D. Blandford, Mon. Not. Royal Astron. Soc. **176**, 465 (1976)
[5] T. Piran, Phys. Rep. **314**, 575 (1998)
[6] J. Hawley, "General Relativistic MHD Simulations of Black Hole Accretion Disks", Proceedings of the 22nd Texas Symposium on Relativistic Astrophysics at Stanford(Stanford California, December 13-17, 2004) Ed. by P.Chen, E. Bloom, G. Madejski, V. Patrosian
[7] J. C. McKinney and C. F. Gammie, Astrophys.J. **611**, 977 (2004)
[8] S.S. Komissarov, Mon. Not. Royal Astron. Soc. **359**, 801 (2005)
[9] H.K. Lee, R.A.M.J. Wijers and G.E. Brown, Phys. Rep. **325**, 83 (2000)
[10] H.K. Lee, G.E. Brown and R.A.M.J. Wijers, Astrophys. J. **536**, 416 (2000)
[11] L.-X. Li, Phys. Rev. **D61**, 084016 (2000)
[12] H.K. Lee and H.K. Kim, J. Korean Phys. Soc. **36**, 188(2000)
[13] L.-X. Li and B. Paczynski, Astrophys. J. **534**, L197 (2000)
[14] D.-X. Wang, W. -H. Lei and R.-Y.Ma, Mon. Not. Royal Astron. Soc. , **342**, 851(2003)
[15] D.A. Uzdensky, Astrophys. J. **620**, 889 (2005)
[16] R. L. Znajek, Mon. Not. Royal Astron. Soc. , **179**, 457 (1977)
[17] S.S. Komissarov, Mon. Not. Royal Astron. Soc. **350**, 427 (2004); astro-ph/0211141
[18] K.S. Thorne, R.H. Price, and D.A. Macdonald, "Black Holes; The Membrane Paradigm" (Yale University Press, New Haven and London, 1986)
[19] R.P. Kerr, Phys. Rev. Lett. **11**, 237 (1963)
[20] R.H. Boyer and R.W. Lindquist, J. Math. Phys. **8**, 265(1967)
[21] T. Damour, Phys. Rev. D **18**, 3598 (1978)
[22] H.K. Lee, Phys. Rev D **64**, 043006(2001)
[23] H.K. Lee, Mod. Phys. Lett. A **21**, 181 (2006)
[24] M.A. Abramowicz, X.-M. Chen, M. Granath and J.-P. Lasota, Astrophys. J. **471**, 762(1996)
[25] C.F. Gammie and R. Popham, Astrophys. J. **498**, 313 (1998)
[26] H. K. Lee, J. Korean Phys. Soc. **40**, 934 (2002)
[27] P. Ghosh and M.A. Abramowicz, Mon. Not. Royal Astron. Soc. **292**, 887 (1997)
[28] M. Livio, G.I. Ogilvie and J.E. Pringle, Astrophys. J. **512**, 100 (1999)
[29] C. Fendt, Astron. Astrophys. **319**, 1025 (1997)
[30] G.V. Ustyugova, R.V.E. Lovelace, M.M. Romanova, H. Li and S.A. Colgate, Astrophys. J. **541**, L21 (2000)
[31] V.S. Beskin, Physics-Uspekhi, **40**, 659 (1997); J. Korean Phys. Soc. **45** S1711 (2004)
[32] D. Macdonald and K.S. Thorne. Mon. Not. Royal Astron. Soc. **198**,345 (1982)
[33] P. Ghosh, Mon. Not. Royal Astron. Soc. **315**, 89 (2000)
[34] C. H. Lee, H. K. Lee and H. Kim, J. Korean Phys. Soc., **43**, 24 (2003)
[35] H.K. Lee and J. Park, Phys. Rev. D **70**, 063001 (2004)
[36] D. Macdonald, Mon. Not. Royal Astron. Soc. **211**, 313 (1984)
[37] S. Shapiro and S.A. Teukolsky,"Black Holes, White Dwarfs, and Neutron Stars" (John Wiley & Sons, New York, 1983)
[38] H.K. Lee and R. Blandford, in preperation(2006)
[39] B. Punsly, "Black Hole Gravitohydrodynamics", Springer Verlag, Berlin, 2001
[40] M. Takahashi, S. Niita, Y.Tatematsu and A. Tomimatsu, Astrophys. J. **363**, 206 (1990)